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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

# ON DETERMINING THE PLANE OF AN ARTIFICIAL SATELLITE'S ORBIT

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LEXINGTON MASSACHUSETTS

## ABSTRACT

The problem of rapidly and accurately determining the orbital plane of an artificial earth satellite from a short series ($\leq 10^{m}$) of observations is posed and solved. The observations can be either angles only data or angles and angular rates. The accuracy is generally 5°. In addition, another look is given to the angular velocity distance estimation technique.

CONTENTS

# I. MOTIVATION

During searches of the equatorial belt one will find known objects and unknown ones. For the known ones immediate identification is desirable so that telescope time can be freed for other purposes. Currently this is done by position only. The use of a spin period, angular velocity, or an orbital plane direction would help. Hence, it was asked whether or not an artificial satellite's orbital plane could be accurately determined in a relatively short ($5 - 10^m$) time.

If an object turned out to be an unknown, then finding it again in the immediate future, so that the telescope could be time shared, was a problem. For low eccentricity, low inclination objects the current dead reckoning tools are adequate. For high eccentricity or high inclination objects these techniques decay in predictive ability as the square of the elapsed time. If one knew the orbital plane then one could constrain the predictions of right ascension and declination to satisfy the orbital plane equation [cf. Eq. (4b)]. Once again a rapid, accurate determination of the orbital plane is desirable.

Finally, there is the problem of recovering unknown satellites the next night. Obviously an along the orbit search needs the orbital plane to proceed. Also, discriminating against several possible initial orbits would be helpful. Hence, the need to determine the orbital plane.

Below we examine the physics and geometry of the problem and test a large number (34) of different artificial satellite orbits based on the assumption they were discovered during equatorial search. The results are generally good to 5°.

## II. THE CONSTRAINT $\underline{r} \cdot \underline{L} = 0$

A unit vector in the direction of the angular momentum vector is given by

$$\underline{L} = (\sin\Omega\sin\iota,\ -\cos\Omega\sin\iota,\ \cos\iota), \tag{1}$$

where $\Omega$ is the longitude of the ascending node on the equator and $\iota$ is the inclination of the orbital plane to the equator. $\Omega\varepsilon[0,360)$, $\iota\varepsilon[0,180)$, and the zero of $\Omega$ is the vernal equinox. The magnitude of the angular momentum vector per unit mass is given by $na^2(1 - e^2)^{1/2}$ where n is the mean motion, a is the semi-major axis, and e is the eccentricity. Since

$$\underline{L} = \underline{r} \times \underline{v}/|\underline{r} \times \underline{v}|, \tag{2}$$

where $\underline{r} = (x,y,z) = r(\cos\delta\cos\alpha, \cos\delta\sin\alpha, \sin\delta)$ is the geocentric location and $\underline{v} = d\underline{r}/dt$ is the geocentric velocity, it follows that

$$\underline{r} \cdot \underline{L} = 0 \text{ and } \underline{v} \cdot \underline{L} = 0. \tag{3}$$

These relationships are true at all instants of time when a, e, $\iota$, and $\Omega$ are interpretated as the elements of the osculating ellipse. Hence, no assumptions about the forces acting on the object (an artificial satellite bound to the earth) need be made except that $d\iota/dt$, $d\Omega/dt$, $da/dt$, and $de/dt$ multiplied by the duration of the observing span be very small. It so happens that for both practical reasons concerned with the mechanics of equatorial search and analytical ones discussed below the observing span

will be on the order of a few minutes. Hence, we can take $\iota$, $\Omega$, a, and e to be constants.

The first part of Eq. (3) can be written as

$$x\sin\Omega\sin\iota - y\cos\Omega\sin\iota + z\cos\iota = 0, \tag{4a}$$

or, in spherical coordinates,

$$r[\cos\delta\tan\iota\sin(\alpha - \Omega) - \sin\delta] = 0. \tag{4b}$$

Note if $\iota = 90°$ the right ascension is always equal to $\Omega$. We also note that the geocentric distance plays no real role in Eq. (4b). In addition, two observations of geocentric position (e.g., $\alpha$ and $\delta$) suffice to determine $\iota$ and $\Omega$, viz,

$$\tan\Omega = \frac{\sin\alpha_1\tan\delta_2 - \sin\alpha_2\tan\delta_1}{\cos\alpha_1\tan\delta_2 - \cos\alpha_2\tan\delta_1}, \tag{5a}$$

$$\tan\iota = \tan\delta_1\csc(\alpha_1 - \Omega) = \tan\delta_2\csc(\alpha_2 - \Omega). \tag{5b}$$

The problem is that we measure topocentric position A, $\Delta$ [the topocentric location is given by $\underline{R} = R(\cos\Delta\cos A, \cos\Delta\sin A, \sin\Delta)$] and not the geocentric values.

Let the observer's geocentric location be given by $\underline{s} = s(\cos\phi'\cos\tau, \cos\phi'\sin\tau, \sin\phi')$ where s is his geocentric distance, $\phi'$ is his geocentric

latitude, and $\tau$ the local sidereal time. Then the relationship between geocentric and topocentric locations is

$$\underline{r} = \underline{R} + \underline{s}. \tag{6}$$

In terms of the measured quantities Eq. (4b) becomes

$$R[\cos\Delta\tan\iota\sin(A - \Omega) - \sin\Delta]$$
$$+ s[\cos\phi'\tan\iota\sin(\tau - \Omega) - \sin\phi'] = 0. \tag{7}$$

We see that the distance does not now drop out. Moreover, if it happens that even within the 5 - $10^{m}$ observing span R changes appreciably, Eq. (7) will not accurately reflect the physics of the problem. Hence, we allow for a constant variation of R, V = dR/dt. Therefore, $\underline{r} \cdot \underline{L} = 0$ becomes

$$f(\iota, \Omega, R, V, t) = (R + Vt)\{\cos\Delta(t)\tan\iota\sin[A(t) - \Omega] -$$
$$\sin\Delta(t)\} + s\{\cos\phi'\tan\iota\sin[\tau(t) - \Omega] - \sin\phi'\} = 0. \tag{8}$$

The full time dependence is now explicitly indicated in Eq. (8).

Presumably we have $N > 3$ measurements of A, $\Delta$, and t, say $\{A_j, \Delta_j, t_j\}$, $j = 1, 2, 3, \ldots, N$. From these and Eq. (8) we wish to determine $\iota$ and $\Omega$ and we apparently get R and V free. Letting $f_j(\iota, \Omega, R, V) = f(\iota, \Omega, R, V, t_j)$ we form

$$F(\iota, \Omega, R, V) = \sum_{j=1}^{N} w_j f_j^2(\iota, \Omega, R, V) \tag{9}$$

and seek the minimum of F with respect to $\iota$, $\Omega$, R, and V. The quantities $w_j$ are the weights of the jth value of f. We argue below for $w_j = 1/N \quad \forall j \varepsilon [1,N]$. The problem is now in the form of a non-linear least squares estimation problem. There exists a standard technique for solving such problems (see the Appendix). Unfortunately, it doesn't work in this case. Instead the method of steepest descent has been successfully used to obtain values of $\iota$, $\Omega$, R, and V (see the Appendix). The accuracy of the results obtained by using Eq. (9) is discussed in §IV.

The weights, $w_j$, are given by

$$1/w_j = (\partial f_j/\partial A_j)^2/w_{A_j} + (\partial f_j/\partial \Delta_j)^2/w_{\Delta_j} + (\partial f_j/\partial t_j)^2/w_{t_j}, \tag{10}$$

where $w_{A_j}$ is the weight of $A_j$, etc. Relative to the weights of topocentric right ascension and declination, the weight of the time is effectively infinite. Also, since $\Delta_j \simeq 0°$ $\forall j \varepsilon [1,N]$, the weights of right ascension and declination are effectively equal. Finally, as $t_N - t_1 \lesssim 10^m$, to first order all the $\partial f_j/\partial A_j$ and $\partial f_j/\partial \Delta_j$ are the same, whence to first order, all of the weights of the same. After reading the Appendix it will become clear that a dependence of the weights on $\iota$, $\Omega$, R, or V is immensely complicating analytically. Hence, even if it weren't true that the

weights are all equal (to first order), I would've taken them to be equal out of computational necessity. This discussion also makes it clear that determining $\iota$, $\Omega$, and R must rest on second order effects so that high accuracy (e.g., better than 1° in $\underline{L}$) is not to be expected. Clearly, determining V rests on third order effects.

## III. THE CONSTRAINT $\underline{v} \cdot \underline{L} = 0$

Unless $\underline{r} \times \underline{v}$ vanishes (e.g., $e = o$) the constraint $\underline{v} \cdot \underline{L} = 0$ is independent of the constraint $\underline{r} \cdot \underline{L} = 0$. Assuming that the acceleration in the topocentric range is negligible, we can write this as

$$\begin{aligned} df(\iota, \Omega, R, V, t)/dt &= V\{\cos\Delta(t)\tan\iota\sin[A(t) - \Omega] - \sin\Delta(t)\} \\ &- (R + Vt)\dot{\Delta}(t)\{\sin\Delta(t) \tan\iota\sin[A(t) - \Omega] + \cos\Delta(t)\} \\ &+ (R + Vt)\dot{A}(t)\cos\Delta(t) \tan\iota\cos[A(t) - \Omega] \\ &+ s\dot{\tau}(t)\cos\phi'\tan\iota\cos[\tau(t) - \Omega] = 0. \end{aligned} \tag{11}$$

The function to be minimized is now

$$F(\iota, \Omega, R, V) = \sum_{j=1}^{N} w_j[f_j^2(\iota, \Omega, R, V) + \dot{f}_j^2(\iota, \Omega, R, V)]. \tag{12}$$

The weights for $f$ and $df/dt$ are taken to be same because the $\dot{f}$ weights are all equal to each other to first order. We note that no new unknowns are introduced in Eq. (12) over those already in Eq. (9).

Solving Eq. (12) is no more difficult than solving Eq. (9) and it has also been performed by the method of steepest descent. If it turns out that no information can be obtained about V even when it's relatively large we merely set $V = 0$ in Eqs. (8 and 11).

## IV. NUMERICAL TESTS

The time spacing for the first series of tests was $t_{j+1} - t_j = 30^s$ $\forall j \varepsilon [1,N]$. This series, using six widely different artificial satellite orbits, included V, used the method of steepest descent, and showed that for a $\approx 5^m$ observing span the direction of $\underline{L}$ could be gotten within a few (5) degrees. Eliminating V from the equations made no difference (it was always 0.000km/sec anyhow) even though $|V|$ was as large as 5km/sec in some cases. I also rewrote Eqs. (8 and 9) using f/R (see the Appendix). For these same satellites and the same observations the results were never better (with or without V) and sometimes poorer. Thus, it seemed important to test a wider variety of satellite orbits, the effect of the angular rates, the effect of the quality of the data, and the effect of the time span of the data.

A total of 34 different satellite orbits have been used to generate topocentric position and topocentric angular velocity. Each has been analyzed (without V) in the following ways; (a) $t_{j+1} - t_j = 30^s$ $\forall j \varepsilon [1,N]$, accuracy in position = $\sqrt{2}''$, N = 11 (e.g., $5^m$), no angular velocity, (b) same as case (a) except N = 21 (e.g., $10^m$), (c) same as case (a) but with a positional accuracy of 9" (SSCSAO quality), (d) same as case (c) except N = 21, (e) same as case (a) except including angular velocities accurate to $0''.1$/sec, and (f) same as case (e) except N = 21.

If w and I are our approximations for $\Omega$ and $\iota$, then let

$$\underline{\ell} = (\sin w \sin I, -\cos w \sin I, \cos I), \qquad (13)$$

and

$$\cos\phi = \underline{\ell} \cdot \underline{L}. \tag{14}$$

If $\underline{\ell}$ and $\underline{L}$ were parallel then $\phi$ would be zero. As there is an ambiguity in determining $\Omega$ from Eq.(5a), we may find a set of values for I and w which has $\underline{\ell}$ anti-parallel to $\underline{L}$. This corresponds to $\phi = 180°$ and the incorrect heliticity. Lastly we note if $\iota$ and I are both small, $\phi$ may be small but $|w - \Omega|$ not necessarily small.

Within a few tenths of a degree in $\phi$ cases (a), (b), (c), and (d) all produce the same results. Table 1 lists the case (c) and case (e) results. The satellites are grouped by increasing mean motion and within each mean motion group by increasing eccentricity. In general adding the angular velocity information produces better results (as would be expected). In a few instances the reverse is true. Moreover, one could presumably find a function of the satellite's orbital elements and the observer's location which would predict the values of $\phi$ in Table 1. Whereas this would explain Table 1, it is obviously of no practical use.

If we concentrate on those satellites with a mean motion of less than 2.5 rev/day the mean value of $\phi$ is $5\overset{\circ}{.}8$. The standard deviation about the mean is $5\overset{\circ}{.}3$. It, therefore, appears that the usefulness of these results is fairly limited. Further experimentation for dead reckoning purposes has been temporarily suspended. Finally, by examining the results for R, it became clear that in the huge majority of the cases what came out was what went in. Hence, a more detailed examination of the angular velocity distance estimation technique seemed in order.

TABLE 1

ACTUAL AND COMPUTED VALUES OF INCLINATION AND THE LONGITUDE OF THE ASCENDING NODE

| | | | | | Case (c) | | | Case (e) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| SDC Number | n (rev/day) | e | $\iota$ (deg) | $\Omega$ (deg) | I (deg) | w (deg) | $\phi$ (deg) | I (deg) | w (deg) | $\phi$ (deg) |
| 3955 | 0.21 | 0.07 | 49.07 | 89.31 | 54.58 | 88.21 | 5.6 | 47.31 | 89.11 | 1.8 |
| 5992 | 0.24 | 0.24 | 21.35 | 44.78 | 23.32 | 45.13 | 2.0 | 20.51 | 45.01 | 0.8 |
| 2766 | 0.22 | 0.74 | 54.87 | 49.86 | 60.30 | 49.49 | 5.4 | 56.32 | 49.72 | 1.4 |
| 10370 | 0.25 | 0.90 | 86.95 | 356.13 | 70.68 | 354.84 | 16.3 | 80.65 | 342.77 | 14.7 |
| 7325 | 0.47 | 0.91 | 81.60 | 273.22 | 98.15 | 273.61 | 16.5 | 166.58 | 247.20 | 93.7 |
| 8062 | 0.65 | 0.69 | 97.40 | 39.66 | 112.72 | 45.68 | 16.4 | 20.58 | 253.39 | 80.2 |
| 83564 | 1.00 | 0.00 | 4.35 | 68.82 | 4.02 | 67.34 | 0.4 | 4.08 | 70.33 | 0.3 |
| 83526 | 1.00 | 0.00 | 0.29 | 92.94 | 0.44 | 187.72 | 0.5 | 0.70 | 34.83 | 0.6 |
| 83580 | 1.00 | 0.00 | 0.62 | 107.64 | 0.12 | 146.15 | 0.5 | 1.24 | 138.65 | 0.8 |
| 83582 | 1.00 | 0.00 | 0.14 | 256.73 | 0.88 | 326.62 | 0.8 | 0.49 | 324.70 | 0.5 |
| 83539 | 1.01 | 0.01 | 0.38 | 270.83 | 0.65 | 246.48 | 0.3 | 0.06 | 245.13 | 0.3 |
| 83504 | 1.00 | 0.00 | 1.74 | 309.92 | 1.04 | 212.97 | 2.1 | 3.18 | 30.42 | 3.4 |
| 83552 | 1.07 | 0.01 | 9.46 | 37.71 | 0.09 | 67.26 | 9.4 | 10.12 | 58.70 | 3.6 |
| 2608 | 1.00 | 0.00 | 9.07 | 44.88 | 0.53 | 67.53 | 8.6 | 9.95 | 66.19 | 3.6 |
| 83517 | 1.02 | 0.11 | 1.60 | 254.15 | 1.34 | 141.83 | 2.4 | 3.58 | 321.79 | 3.3 |
| 83589 | 1.02 | 0.13 | 6.59 | 178.47 | 4.05 | 316.90 | 10.0 | 7.70 | 144.57 | 4.3 |
| 83544 | 1.02 | 0.14 | 6.82 | 332.94 | 2.27 | 350.11 | 4.7 | 7.98 | 351.89 | 2.7 |
| 73505 | 1.02 | 0.14 | 14.17 | 94.05 | 4.17 | 91.03 | 10.0 | 14.78 | 105.74 | 3.0 |
| 10637 | 1.00 | 0.24 | 28.42 | 201.65 | 32.49 | 201.33 | 4.1 | 28.85 | 198.30 | 1.7 |
| 2805 | 1.23 | 0.82 | 51.83 | 212.34 | 50.79 | 211.27 | 1.3 | 49.14 | 210.80 | 2.9 |
| 748 | 1.06 | 0.77 | 66.98 | 1.88 | 70.51 | 1.06 | 3.6 | 74.43 | 8.84 | 9.9 |
| 10684 | 2.00 | 0.00 | 63.26 | 219.64 | 62.62 | 218.86 | 0.9 | 63.93 | 226.35 | 6.1 |
| 9864 | 2.32 | 0.72 | 24.57 | 115.41 | 14.84 | 121.46 | 9.9 | 22.49 | 121.99 | 3.4 |
| 10857 | 2.33 | 0.73 | 27.33 | 334.65 | 38.81 | 340.59 | 11.9 | 37.82 | 356.45 | 15.6 |
| 9892 | 2.01 | 0.71 | 65.68 | 22.40 | 65.52 | 22.45 | 0.2 | 62.46 | 23.49 | 3.4 |
| 10984 | 2.01 | 0.74 | 63.10 | 86.42 | 68.51 | 88.11 | 5.6 | 64.96 | 92.06 | 5.4 |
| 83195 | 2.07 | 0.73 | 62.43 | 139.69 | 58.16 | 137.87 | 4.6 | 78.57 | 167.93 | 31.0 |
| 9411 | 1.97 | 0.72 | 63.91 | 269.22 | 51.12 | 271.05 | 12.9 | 58.16 | 258.33 | 11.1 |
| 83201 | 2.00 | 0.73 | 62.77 | 248.54 | 62.76 | 248.88 | 0.3 | 59.81 | 251.59 | 4.0 |
| 11027 | 2.81 | 0.69 | 31.26 | 85.33 | 21.51 | 288.30 | 51.7 | 18.63 | 120.41 | 19.0 |
| 8599 | 2.99 | 0.02 | 124.62 | 198.38 | 128.19 | 190.88 | 7.0 | 124.69 | 194.89 | 2.9 |
| 10276 | 3.65 | 0.63 | 46.62 | 98.19 | 21.18 | 92.14 | 25.6 | 33.48 | 104.58 | 13.8 |
| 2529 | 4.65 | 0.39 | 32.19 | 302.97 | 36.23 | 297.75 | 5.0 | 32.71 | 292.50 | 5.6 |
| 16 | 10.42 | 0.21 | 34.26 | 2.33 | 30.90 | 54.63 | 27.6 | 18.86 | 346.20 | 16.9 |

## V. THE DISTANCE ESTIMATION TECHNIQUE*

As this is the largest body of data I have yet had with which to test the angular velocity distance estimation technique, I performed such an analysis. Figure 1 is a plot of the estimated distance against the true distance (both are geocentric). Low eccentricity orbits are plotted as dots, high eccentricity orbits as plus signs. The two lines represent $r_{est} = r$ and $r_{est} = 0.83r$. The latter includes the correction of the unknown eccentricity assuming a uniform distribution of eccentricities in the artificial satellite population. Once beyond $\simeq$ 3 earth radii the technique works quite well. Of course, for all the small eccentricity objects $r_{est}$ is systematically too low by 1 - 0.83 = 17%. To remedy this, since about 75% of the total deep space artificial satellite population is low eccentricity, we can split the difference (e.g., use 0.91 instead of 0.83) or use a two point eccentricity distribution. The latter, if 75% are near e = 0 and 25% near e = 0.7 yields 0.94. In the future we'll split the difference.

*See §VIIA of L. G. Taff, "Astrometry in Small Fields," Technical Note 1977-2, Lincoln Laboratory, M.I.T. (14 June 1977), DDC AD-A043568/5.

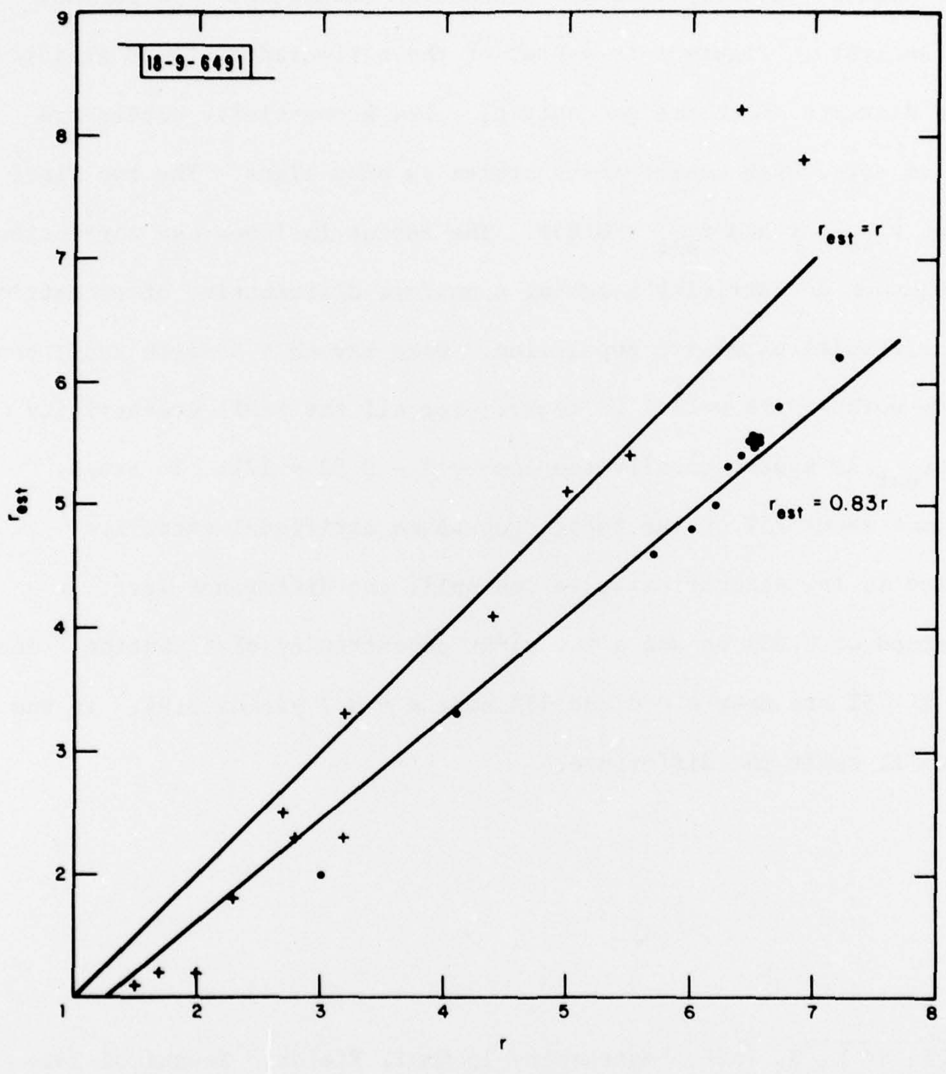


Fig. 1. Plot of estimated geocentric distance ($r_{est}$) vs the actual geocentric distance (r). Plus signs denote high eccentricity objects. Dots denote low eccentricity objects.

APPENDIX

In this section I want to review the standard technique for solving a non-linear estimation problem, review the method of steepest descent, and examine the application of these techniques to Eqs. (9 and 12). Suppose we have some function g of the unknown parameters $\underline{x}$ and we wish to minimize

$$G = \sum_{j=1}^{N} w_j g_j^2(\underline{x}). \tag{A1}$$

The standard technique to find the value of $\underline{x}$, say $\underline{x}_m$, that minimizes G is to expand $g_j(\underline{x})$ about some guess for $\underline{x}_m$, say $\underline{x}_o$. Then, to first order,

$$G = \sum_{j=1}^{N} w_j [g_j(\underline{x}_o) + \nabla_{\underline{x}} g_j(\underline{x})|_{\underline{x} = \underline{x}_o} \cdot (\underline{x} - \underline{x}_o)]^2. \tag{A2}$$

G, as given by Eq. (A2) is now a linear function of $\underline{x} - \underline{x}_o$ and we form the normal equations as in the ordinary linear case (e.g., demand $\nabla_{\underline{x}} G = 0$). After solving the normal equations for $\underline{x} - \underline{x}_o$ our new guess for $\underline{x}_m$ is $\underline{x}_n = (\underline{x} - \underline{x}_o) + \underline{x}_o$. If necessary this iteration is repeated until $|\underline{x} - \underline{x}_o|$ is sufficiently small and $|\nabla_{\underline{x}} G|$ is sufficiently small too.

This procedure won't work for Eqs. (8 and 9) because of the following: It is logical to use the geocentric relationships, Eqs. (5), to obtain guesses for $\iota$ and $\Omega$. We can guess R by the angular velocity technique and V = 0 should do. The matrix of the normal equations, along the main diagonal, will then have terms proportional to N $[\cos\Delta \tan\iota \sin(A_j - \Omega) - \sin\Delta_j]^2$ and $(t_N - t_1)^2$ times this. Clearly each of these is very small,

the second being $10^{-5}$ of the first. Hence, inversion of the matrix is numerically impossible. One might think that this could be avoided by using f/R instead of f in Eq. (9). The present difficulty would be removed but a new one introduced. As $s/R \simeq 1/6$ for the satellites of interest, the matrix of the normal equations now contains terms of the order of $N(s/R^2)^2$ and $N(t_N - t_1)^2(s/R^2)^2$ on the main diagonal. These are of the order of $10^{-2}$ and $10^{-7}$, respectively. Hence, numerical inversion is again impossible.

Having exhausted analytically rearranging Eqs. (8 and 9) we turn to a completely different technique. Let us return to Eqs. (A1 and A2) but continue the expansion of $G(\underline{x} + \lambda\underline{X})$ to second order terms. If $\Gamma(\underline{x})$ is the Hessian matrix of G then

$$G(\underline{x} + \lambda\underline{X}) = G(\underline{x}) + \nabla_{\underline{x}}G(\underline{x}) \cdot \lambda\underline{X} + (\lambda^2/2)\,\underline{X} \cdot \Gamma(\underline{x}) \cdot \underline{X}. \tag{A3}$$

As we are looking for a minimum of G we take $\underline{X}$ to be in the direction of the maximum rate of decrease of G, i.e.,

$$\underline{X} = - \nabla_{\underline{x}}G(\underline{x}). \tag{A4}$$

With this value for $\underline{X}$ we can find the value of $\lambda$ to use by insisting that $G[\underline{x} - \lambda\nabla_{\underline{x}}G(\underline{x})]$ be a minimum with respect to $\lambda$. The result is

$$\lambda = |\nabla_{\underline{x}}G(\underline{x})|^2/[\nabla_{\underline{x}}G(\underline{x}) \cdot \Gamma(\underline{x}) \cdot \nabla_{\underline{x}}G(\underline{x})]. \tag{A5}$$

Thus, starting from a guess $\underline{x} = \underline{x}_o$, for $\underline{x}_m$, we compute our new guess from

$$\underline{x}_n = \underline{x}_o - \lambda \nabla_{\underline{x}} G(\underline{x})\Big|_{\underline{x} = \underline{x}_o} \tag{A6}$$

with $\lambda$ evaluated via Eq. (A5) at $\underline{x} = \underline{x}_o$. If $G(\underline{x})$ has a unique minimum (e.g., $\underline{x}_m$) in any closed region and the metric defined by $\Gamma(\underline{x})$ has a positive upper bound in this region then this iteration does converge to $\underline{x}_m$. For most practical problems this technique is difficult to apply because one needs accurate values for $\Gamma(\underline{x})$.

For the problem posed by Eqs. (8 and 9), or more generally Eqs. (11 and 12), all of the second derivatives can be computed. In particular, for Eqs. (11 and 12),

$$\partial F/\partial \iota = 2 \sum_{j=1}^{N} w_j [f_j (\partial f_j/\partial \iota) + \dot{f}_j (\partial \dot{f}_j/\partial \iota)], \tag{A7}$$

etc., and

$$\partial^2 F/\partial \iota^2 = 2 \sum_{j=1}^{N} w_j [(\partial f_j/\partial \iota)^2 + f_j (\partial^2 f_j/\partial \iota^2) + (\partial \dot{f}_j/\partial \iota)^2 + \dot{f}_j (\partial^2 \dot{f}_j/\partial \iota^2)], \tag{A8a}$$

$$\partial^2 F/\partial \iota \partial \Omega = 2 \sum_{j=1}^{N} w_j [\partial f_j/\partial \iota)(\partial f_j/\partial \Omega) + f_j (\partial^2 f_j/\partial \iota \partial \Omega) + (\partial \dot{f}_j/\partial \iota)(\partial \dot{f}_j/\partial \Omega)$$

$$+ \dot{f}_j (\partial^2 \dot{f}_j/\partial \iota \partial \Omega)], \tag{A8b}$$

etc.

The needed derivatives of $f_j$ and $df_j/dt$ are given by

$$\partial f_j/\partial \iota = \sec^2\iota[(R + Vt_j)\cos\Delta_j \sin(A_j - \Omega) + s\cos\phi' \sin(\tau_j - \Omega)], \quad \text{(A9a)}$$

$$\partial f_j/\partial \Omega = -\tan\iota[(R + Vt_j)\cos\Delta_j \cos(A_j - \Omega) + s\cos\phi' \cos(\tau_j - \Omega)], \quad \text{(A9b)}$$

$$\partial f_j/\partial R = \cos\Delta_j \tan\iota \sin(A_j - \Omega) - \sin\Delta_j, \quad \text{(A9c)}$$

$$\partial f_j/\partial V = t_j \partial f_j/\partial R, \quad \text{(A9d)}$$

$$\partial \dot{f}_j/\partial \iota = \sec^2\iota[V\cos\Delta_j \sin(A_j - \Omega) - (R + Vt_j)\dot{\Delta}_j \sin\Delta_j \sin(A_j - \Omega)$$

$$+ (R + Vt_j)\dot{A}_j \cos\Delta_j \cos(A_j - \Omega) + s\dot{\tau}_j \cos\phi' \cos(\tau_j - \Omega)], \quad \text{(A10a)}$$

$$\partial \dot{f}_j/\partial \Omega = -\tan\iota[V\cos\Delta_j \cos(A_j - \Omega) - (R + Vt_j)\dot{\Delta}_j \sin\Delta_j \cos(A_j - \Omega)$$

$$- (R + Vt_j)\dot{A}_j \cos\Delta_j \sin(A_j - \Omega) - s\dot{\tau}_j \cos\phi' \sin(\tau_j - \Omega)], \quad \text{(A10b)}$$

$$\partial \dot{f}_j/\partial R = -\dot{\Delta}_j[\sin\Delta_j \tan\iota \sin(A_j - \Omega) + \cos\Delta_j]$$

$$+ \dot{A}_j \cos\Delta_j \tan\iota \cos(A_j - \Omega), \quad \text{(A10c)}$$

$$\partial \dot{f}_j/\partial V = \partial f_j/\partial R + t_j \partial \dot{f}_j/\partial R, \quad \text{(A10d)}$$

$$\partial^2 f_j/\partial\iota^2 = 2\tan\iota\,\partial f_j/\partial\iota, \tag{A11a}$$

$$\partial^2 f_j/\partial\iota\partial\Omega = \sec^2\iota\cot\iota\,\partial f_j/\partial\Omega, \tag{A11b}$$

$$\partial^2 f_j/\partial\iota\partial R = \sec^2\iota\cos\Delta_j\sin(A_j - \Omega), \tag{A11c}$$

$$\partial^2 f_j/\partial\iota\partial V = t_j\partial^2 f_j/\partial\iota\partial R, \tag{A11d}$$

$$\partial^2 f_j/\partial\Omega^2 = -\cos^2\iota\tan\iota\,\partial f_j/\partial\iota, \tag{A11e}$$

$$\partial^2 f_j/\partial\Omega\partial R = -\tan\iota\cos\Delta_j\cos(A_j - \Omega), \tag{A11f}$$

$$\partial^2 f_j/\partial\Omega\partial V = t_j\partial^2 f_j/\partial\Omega\partial R, \tag{A11g}$$

$$\partial^2 f_j/\partial R^2 = \partial^2 f_j/\partial R\partial V = \partial^2 f_j/\partial V^2 = 0, \tag{A11h}$$

$$\partial^2\dot{f}_j/\partial\iota^2 = 2\tan\iota\,\partial\dot{f}_j/\partial\iota, \tag{A12a}$$

$$\partial^2\dot{f}_j/\partial\iota\partial\Omega = \sec^2\iota\cot\iota\,\partial\dot{f}_j/\partial\Omega, \tag{A12b}$$

$$\partial^2\dot{f}_j/\partial\iota\partial R = \sec^2\iota[-\dot{\Delta}_j\sin\Delta_j\sin(A_j - \Omega) + \dot{A}_j\cos\Delta_j\cos(A_j - \Omega)] \tag{A12c}$$

$$\partial^2\dot{f}_j/\partial\iota\partial V = \partial^2 f_j/\partial\iota\partial R + t_j\partial^2\dot{f}_j/\partial\iota\partial R, \tag{A12d}$$

$$\partial^2 \dot{f}_j/\partial\Omega^2 = -\cos^2 \iota \tan \iota \,\partial \dot{f}_j/\partial \iota, \tag{A12e}$$

$$\partial^2 \dot{f}_j/\partial\Omega\partial R = \tan\iota[\dot{\Delta}_j \sin\Delta_j \cos(A_j - \Omega) + \dot{A}_j \cos\Delta_j \sin(A_j - \Omega)], \tag{A12f}$$

$$\partial^2 \dot{f}_j/\partial\Omega\partial V = \partial^2 f_j/\partial\Omega\partial R + t_j \partial^2 \dot{f}_j/\partial\Omega\partial R, \tag{A12g}$$

$$\partial^2 \dot{f}_j/\partial R^2 = \partial^2 \dot{f}_j/\partial R\partial V = \partial^2 \dot{f}_j/\partial V^2 = 0. \tag{A12h}$$

Since the V derivatives are, in general, a factor of $t_j \simeq 0.007$ down from the R derivatives we expect poor accuracy for the final value of V.

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| ETS | angles only data |
| GEODSS | angles and angular rates |
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

< or = 10 superscript m

The problem of rapidly and accurately determining the orbital plane of an artificial earth satellite from a short series ($\leqslant 10^m$) of observations is posed and solved. The observations can be either angles only data or angles and angular rates. The accuracy is generally 5°. In addition, another look is given to the angular velocity distance estimation technique.

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